On the Complexity of Intersecting Regular, Context-free, and Tree Languages

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Fixed Levels

- Let two parameterized problems X and Y with parameters p_1 and p_2, respectively, be given.
- Consider fixing the parameters p_1 and p_2 to a fixed value k.
- We get fixed levels X_k and Y_k, respectively.
- We can expand out problems X and Y into infinitely many fixed levels.
- As a result, we get infinite families $\{X_k\}_{k \in \mathbb{N}}$ and $\{Y_k\}_{k \in \mathbb{N}}$.

fpt-Reductions

- In parameterized complexity theory, there are existing notions of *fpt*-reduction and *fpt*-equivalence.
- In our work, we are focused on the fixed levels of the parameterized problems.
- We are especially interested in the case where the fixed levels of X and Y exactly match up.
- Intuitively, we want X_k is reducible to Y_k. We don't merely want a polynomial p such that X_k is reducible to $Y_{p(k)}$.

Level-By-Level Reductions

Definition

$\{X_k\}_{k\in\mathbb{N}}$ is LBL-reducible to $\{Y_k\}_{k\in\mathbb{N}}$ if there is an infinite family of reductions $\{r_k\}_{k\in\mathbb{N}}$ such that for each k, r_k reduces X_k to Y_k and each reduction r_k is computable in some fixed polynomial amount of time.

- More formally, there needs to exist a fixed function f and a constant c such that each r_k is computable in $f(k) \cdot n^c$ time.
- $\{X_k\}_{k\in\mathbb{N}}$ and $\{Y_k\}_{k\in\mathbb{N}}$ are LBL-equivalent if both families are LBL-reducible to each other.

The Problem

Intersection Non-Emptiness for DFA's

Given a finite list of DFA's, is there a string that simultaneously satisfies all the DFA's?

- We denote this problem by $\mathrm{IE}_{\mathcal{D}}$.
- We use n for the total length of the input's encoding.
- We use k for the number of DFA's in the list.

An Acceptance Problem

Acceptance for logspace bounded NTM's

Given an encoding of a NTM M, an input string x, and a number k, does M accept x using at most $k\log(n)$ bits of memory?

- We denote this problem by $N^{S}_{O(\log(n))}$.
- The TM's that we consider have a two-way read only input tape and a two-way read/write binary work tape.

Equivalence

- Consider expanding out $\mathrm{IE}_{\mathcal{D}}$ into infinitely many fixed levels based on the number of DFA's.
- We get the infinite family $\{k\text{-}\mathrm{IE}_{\mathcal{D}}\}_{k \in \mathbb{N}}$.
- Consider expanding out $N^S_{O(\log(n))}$ into infinitely many fixed levels based on the coefficient.
- We get the infinite family $\{N^S_{k \log(n)}\}_{k \in \mathbb{N}}$.
- These two infinite families are LBL-equivalent (ICALP 2014).

Equivalence

Theorem

$\{N^S_{k\log(n)}\}_{k\in\mathbb{N}}$ is LBL-reducible to $\{k\text{-IE}_{\mathcal{D}}\}_{k\in\mathbb{N}}$.

- Let a $k\log(n)$-space bound NTM M be given.
- Let an input string x of length n be given.
- A computation of M on x is a sequence of configurations.
- Each configuration includes the tape content.

Equivalence

Theorem

$\{N^S_{k\log(n)}\}_{k\in\mathbb{N}}$ is LBL-reducible to $\{k\text{-IE}_{\mathcal{D}}\}_{k\in\mathbb{N}}$.

- The tape is a sequence of $k\log(n)$ bits.
- We can break up this sequence into k regions.
- Each region will cover $\log(n)$ bits of the tape.
- We build k DFA's to collectively verify a "computation".
- We assign one DFA to each region.

The Problem

Intersection Non-Emptiness for DFA's and One PDA

Given a finite list of DFA's and one PDA, is there a string that simultaneously satisfies all of the automata?

- We denote this problem by $\mathrm{IE}_{\mathcal{P}}$.
- We use n for the total length of the input's encoding.
- We use k for the number of DFA's in the list.

An Acceptance Problem

Acceptance for logspace bounded AuxPDA's

Given an encoding of an auxiliary PDA M, an input string x, and a number k, does M accept x using at most $k \log(n)$ bits of auxiliary memory?

- We denote this problem by $Aux^{S}_{O(\log(n))}$.
- An auxiliary PDA has a two-way read only input tape, a stack, and a two-way read/write auxiliary binary work tape.

Equivalence

- Consider expanding out $\mathrm{IE}_{\mathcal{P}}$ into infinitely many fixed levels based on the number of DFA's.
- We get the infinite family $\{k\text{-}\mathrm{IE}_{\mathcal{P}}\}_{k \in \mathbb{N}}$.

- Consider expanding out $Aux^{S}_{O(\log(n))}$ into infinitely many fixed levels based on the coefficient.
- We get the infinite family $\{Aux^{S}_{k\log(n)}\}_{k \in \mathbb{N}}$.

- These two infinite families are LBL-equivalent.

Equivalence

Theorem

$\{Aux^{S}_{k\log(n)}\}_{k\in\mathbb{N}}$ is LBL-reducible to $\{k\text{-IE}_{\mathcal{P}}\}_{k\in\mathbb{N}}$.

- The reduction is essentially the same.
- We build k DFA's and one PDA to collectively verify an AuxPDA "computation".
- The auxiliary work tape is split up into k regions.
- We assign one DFA to each region.
- The single PDA is used to keep track of the stack.

Another Equivalence

- We also look at intersection non-emptiness for tree automata.
- Using similar techniques, we prove LBL-equivalence with acceptance for logspace bounded ATM's.
- We get $\{k\text{-IE}_{\mathcal{T}}\}_{k \in \mathbb{N}}$ and $\{A^S_{k \log(n)}\}_{k \in \mathbb{N}}$ are LBL-equivalent.

Recap

- Solving IE for k DFA's is equivalent to simulating a NTM that uses $k \log(n)$ bits of memory.
- Solving IE for k DFA's and one PDA is equivalent to simulating an AuxPDA that uses $k \log(n)$ bits of memory.
- Solving IE for k Tree Automata is equivalent to simulating an ATM that uses $k \log(n)$ bits of memory.

An Acceptance Problem

Acceptance for polytime bounded DTM's

Given an encoding of a DTM M, an input string x, and a number k, does M accept x in n^k steps or less?

- We denote this problem by $D^T_{\text{poly}(n)}$.
- Consider expanding out $D^T_{\text{poly}(n)}$ into infinitely many fixed levels based on the exponent.
- We get the infinite family $\{D^T_{n^k}\}_{k \in \mathbb{N}}$.

Applying Known Results

Known Results

The following machine models are equivalent:

1. n^k-time bound DTM's
2. $k\log(n)$-space bounded AuxPDA's
3. $k\log(n)$-space bounded ATM's.

- Therefore, the following are LBL-equivalent: $\{D^T_{n^k}\}_{k\in\mathbb{N}}$, $\{Aux^S_{k\log(n)}\}_{k\in\mathbb{N}}$, and $\{A^S_{k\log(n)}\}_{k\in\mathbb{N}}$.
- Further, the following are LBL-equivalent: $\{D^T_{n^k}\}_{k\in\mathbb{N}}$, $\{k\text{-IE}_{\mathcal{P}}\}_{k\in\mathbb{N}}$, and $\{k\text{-IE}_{\mathcal{T}}\}_{k\in\mathbb{N}}$.

Lower Bounds

- By the time hierarchy theorem, we get an n^k-time lower bound for the $D^T_{n^k}$ problem.
- Since LBL-equivalence preserves lower bounds, we get the following:
  - $\exists c_1 \, \forall k \;\; k\text{-IE}_{\mathcal{P}} \notin \text{DTIME}(n^{c_1 k})$
  - $\exists c_2 \, \forall k \;\; k\text{-IE}_{\mathcal{T}} \notin \text{DTIME}(n^{c_2 k})$
- Similarly, we prove lower bounds for non-emptiness for multi-stack PDA's with $\log(k)$ phase switches.
- These are near tight lower bounds for each level of the respective parameterized problem.