Creating Patterns with Distance Functions & Voronoi Diagrams

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Motivation

- Voronoi Diagrams are a classic way of dividing a space based on seed points.
  - Select seed points
  - Create regions based on which seed point is closest
  - Euclidean distance is commonly used
- **Question**

  What if we use a different distance function?

Distance Functions

- We implemented a drawing algorithm that we called "Voronoi" using the AlgoArt Platform (algoart.org).
- Using this algorithm, we generated 500+ Voronoi Diagrams with 15 unique distance functions broken down into the following three categories.

Classical Functions

- Euclidean, Manhattan, Hyperbolic

Experimental Functions

- AbsDiff, Chaos 1 & 2, DiffProd, Euclihattan, MinDiff, Odd, Wave

Polar Functions

- Polar variants of Euclidean, Manhattan, Hyperbolic, Wave

Note: Our experimental functions relax the positivity and triangle inequality requirements of metrics.

Results

Using non-classical distance functions leads to a variety of artistic and practical use cases. In particular, the resulting Voronoi Diagrams create:

- Unconventional tilings and novel artistic designs
- Repeated patterns with cultural analogs (e.g. textile patterns)
- Region boundaries that emphasize irregular shapes (e.g. curves that resemble fluids)

Gallery

AlgoArt's Digital Gallery displays 500+ of our Voronoi Diagrams with the parameters that were used to generate them.

Git Repo

AlgoArt's Open Source Creator Studio includes the source code for our Voronoi Diagram drawing algorithm.

Acknowledgements

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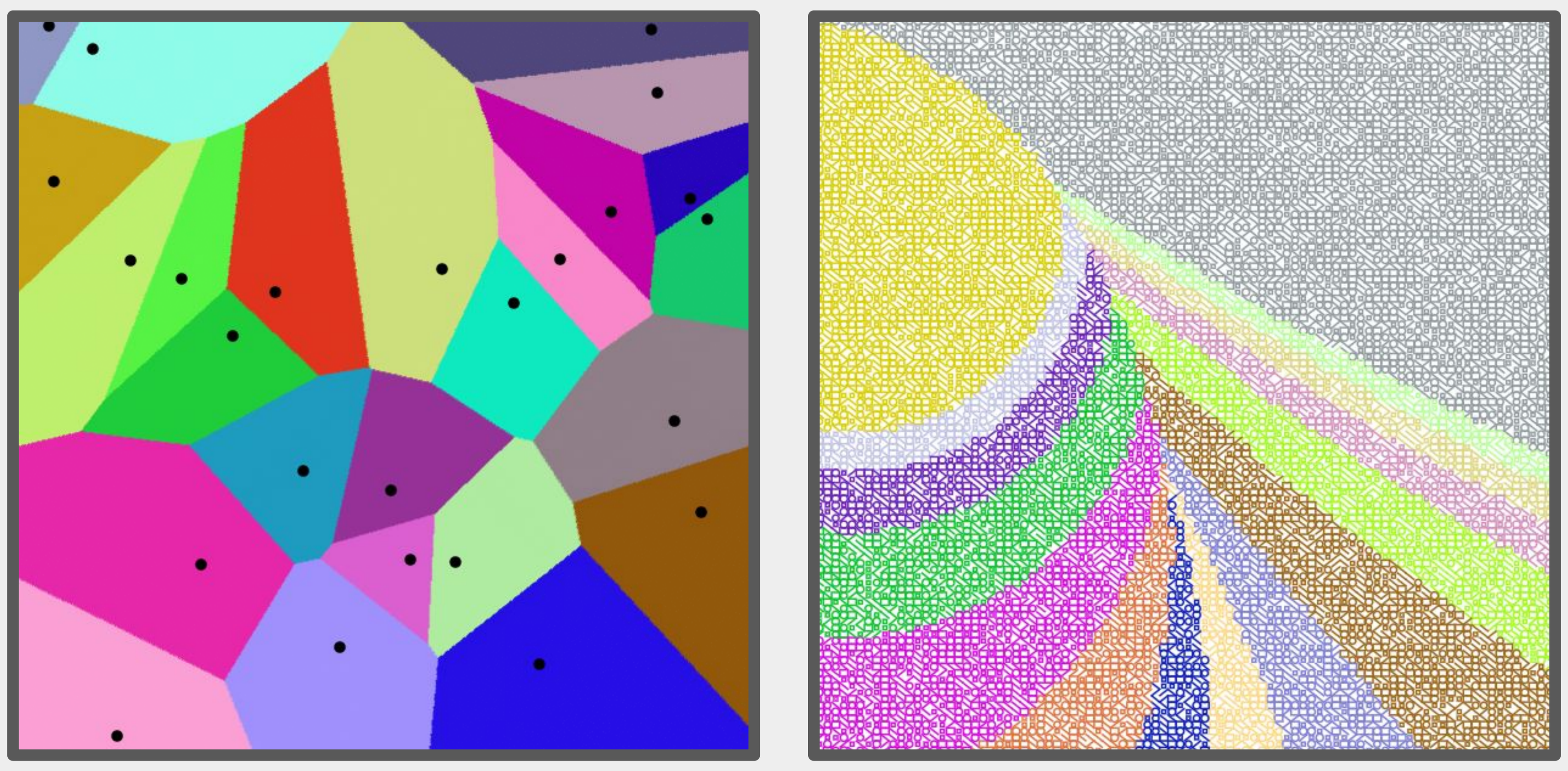
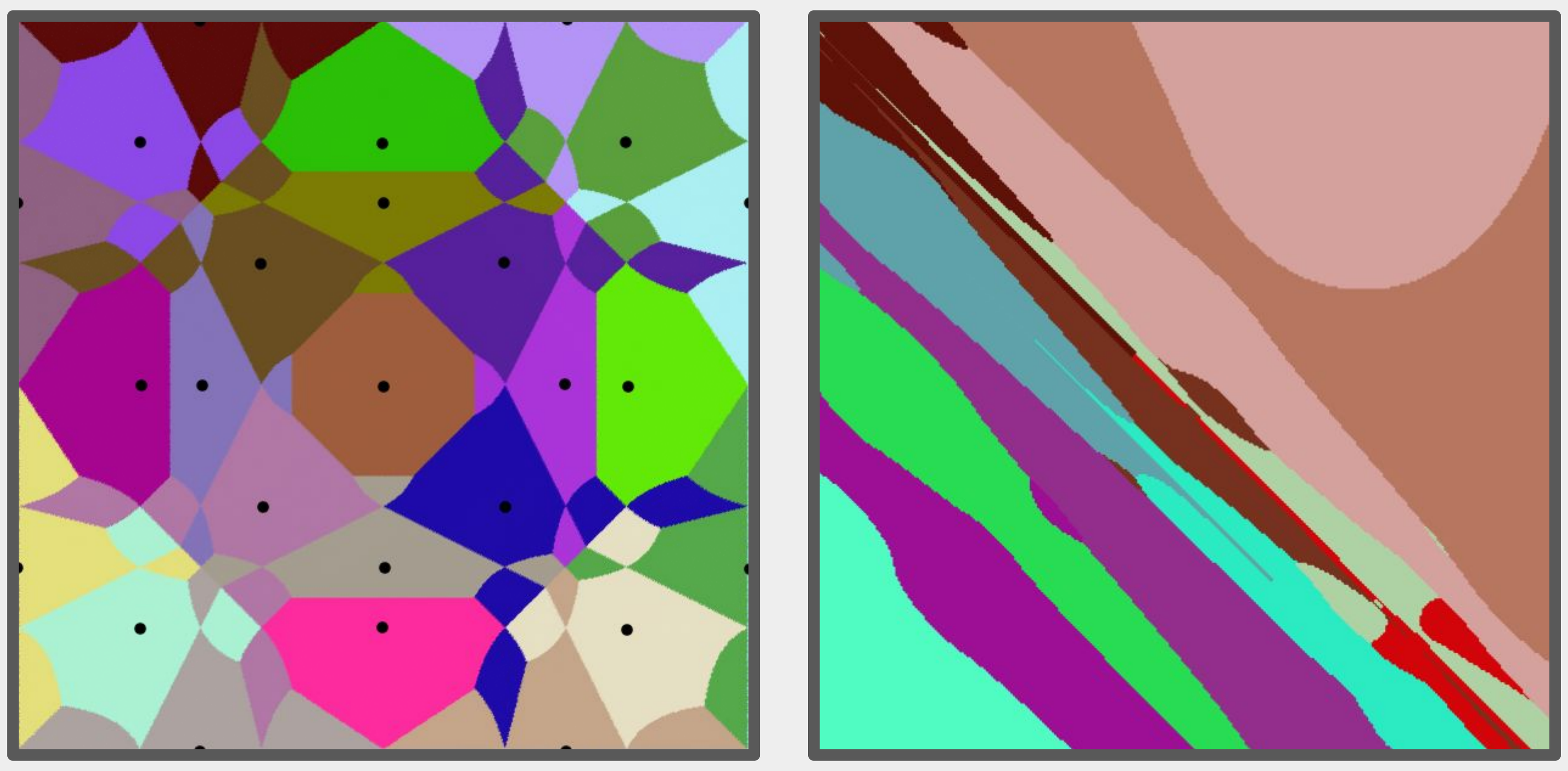
Voronoi Diagram Examples

Classical Distance Functions



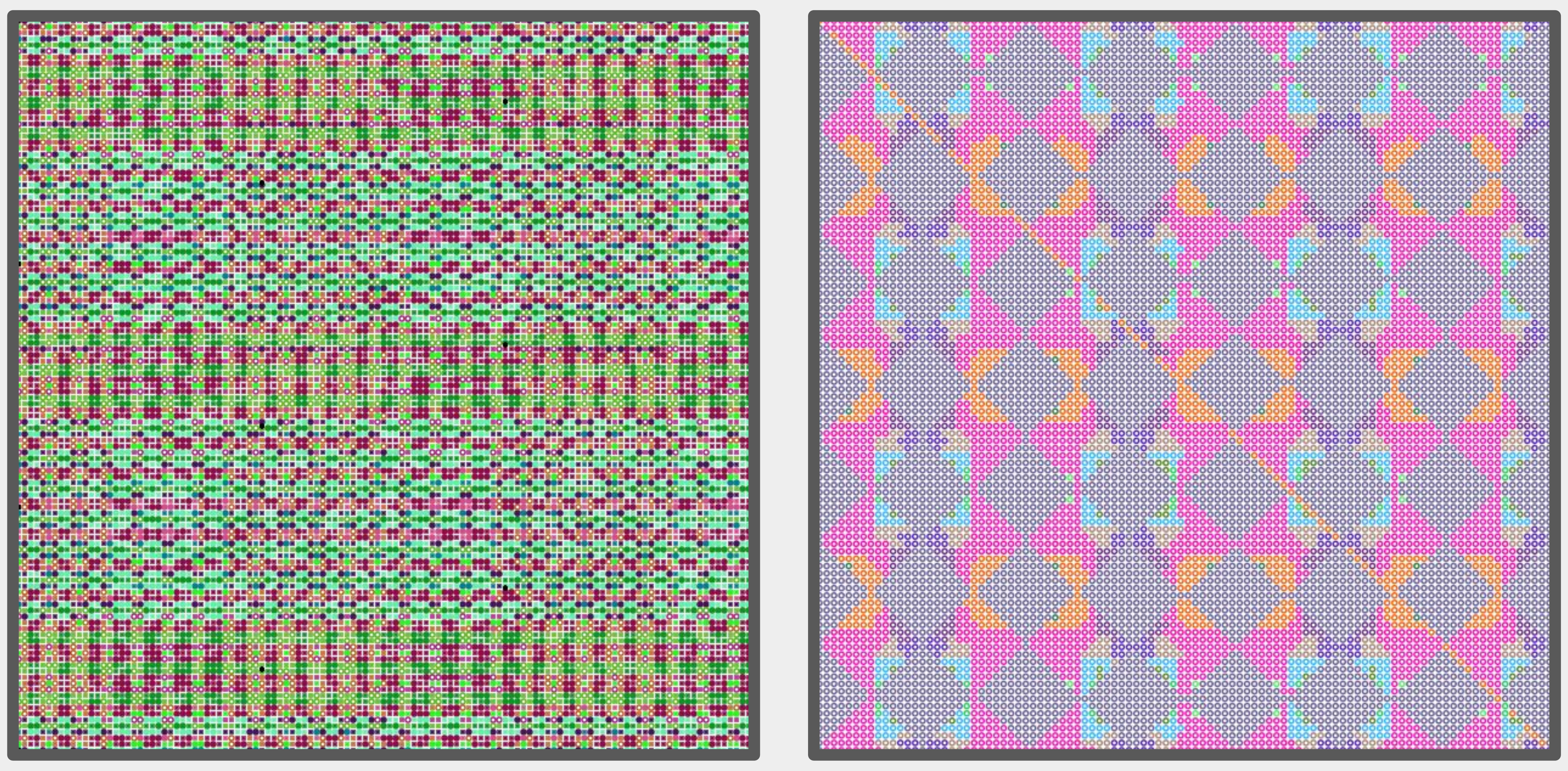
Euclidean (left) and Hyperbolic (right)

Experimental Distance Functions



DiffProd (left) and Wave (right)

Polar Distance Functions



Polar Euclidean (left) and Polar Hyperbolic (right)

Non-Classical Distance Functions

Euclihattan

$$\left| |x_2 - x_1| + |y_2 - y_1| + \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2} \right|$$

PolarEuclidean

$$\sqrt{(cos(x_2) - cos(x_1))^2 + (sin(y_2) - sin(y_1))^2}$$

PolarManhattan

$$|cos(x_2) - cos(x_1)| + |sin(y_2) - sin(y_1)|$$

PolarHyperbolic

$$arcosh\left(1 + 2\frac{(cos(x_2)-cos(x_1))^2 + (sin(y_2)-sin(y_1))^2}{(1-(cos(x_1)^2+sin(y_1)^2))(1-(cos(x_2)^2+sin(y_2)^2))}\right)$$

Wave

$$\left| \sqrt{|x_2 - x_1|} \cdot \frac{x_1 - x_2}{w/2} - \sqrt{|y_2 - y_1|} \cdot \frac{y_1 - y_2}{h/2} \right|$$

PolarWave

$$\left| \sqrt{|cos(x_2) - cos(x_1)|} \cdot \frac{cos(x_1)-cos(x_2)}{w/2} - \sqrt{|sin(y_2) - sin(y_1)|} \cdot \frac{sin(y_1)-sin(y_2)}{h/2} \right|$$

MinDiff

$$min(|x_2 - x_1|, |y_2 - y_1|)$$

AbsDiff

$$\left| |x_2 - x_1| - |y_2 - y_1| \right|$$

DiffProd

$$|x_2 - x_1| \cdot |y_2 - y_1|$$

Chaos

$$\left| \sqrt{|x_2 - x_1|} \cdot (x_1 + x_2)/2w - \sqrt{|y_2 - y_1|} \cdot (y_1 + y_2)/2h \right|$$

Odd

$$|y_2 - y_1| + 2w/75 * \sqrt{|x_2 - x_1|}$$

Chaos2

$$\left| |y_2 - y_1| - 2w/75 * \sqrt{|x_2 - x_1|} \right|$$